### **UNIT 3: CO-ORDINATE GEOMETRY**



## **Syllabus**

> Review : Concepts of co-ordinate geometry, graphs of linear equations. Distance formula. Section formula (internal division). Area of a triangle.

## **Trend Analysis**

	20	18	2019		2020	
List of Concepts	Delhi	Outside Delhi	Delhi	Outside Delhi	Delhi	Outside Delhi
Distance between two points and section formula	1 Q (2 M) 1 Q (3 M)		1 Q (1 M) 1 Q (3 M)	1 Q (3 M)	2 Q (1 M) 1 Q (3 M)	3 Q (1 M) 1 Q (3 M) 1 Q (4 M)
Area of a Triangle	1 Q (3 M)		2 Q (3 M)		1 Q (1 M) 1 Q (3 M)	1 Q (3 M)



Revision Notes

Two perpendicular number lines intersecting at origin are called co-ordinate axes. The horizontal line is the X-axis (denoted by X'OX) and the vertical line is the Y-axis (denoted by Y'OY).



- > The point of intersection of X-axis and Y-axis is called origin and denoted by O.
- Cartesian plane is a plane obtained by putting the co-ordinate axes perpendicular to each other in the plane. It is also called co-ordinate plane or XY-plane.
- > The *x*-co-ordinate of a point is its perpendicular distance from Y-axis.
- > The *y*-co-ordinate of a point is its perpendicular distance from X-axis.
- ▶ The point where the X-axis and the Y-axis intersect has co-ordinate point (0, 0).
- > The abscissa of a point is the *x*-co-ordinate of the point.
- > The ordinate of a point is the *y*-co-ordinate of the point.
- > If the abscissa of a point is x and the ordinate of the point is y, then (x, y) is called the co-ordinates of the point.
- The axes divide the Cartesian plane into four parts called the quadrants (one fourth part), numbered I, II, III and IV anti-clockwise from OX.
- The co-ordinates of a point on the X-axis are of the form (x, 0) and that of the point on Y-axis are (0, y).
- Sign of co-ordinates depicts the quadrant in which it lies. The co-ordinates of a point are of the form (+, +) in the first quadrant, (-, +) in the second quadrant, (-, -) in the third quadrant and (+, -) in the fourth quadrant.
- Three points A, B and C are collinear if the distances AB, BC and CA are such that the sum of two distances is equal to the third.
- > Three points A, B and C are the vertices of an equilateral triangle if AB = BC = CA.
- > The points A, B and C are the vertices of an isosceles triangle if AB = BC or BC = CA or CA = AB.
- > Three points A, B and C are the vertices of a right triangle, if  $AB^2 + BC^2 = CA^2$ .



➢ For the given four points A, B, C and D :



- 1. If AB = BC = CD = DA; AC = BD, then ABCD is a square.
- 2. If AB = BC = CD = DA;  $AC \neq BD$ , then ABCD is a rhombus.
- 3. If AB = CD, BC = DA; AC = BD, then ABCD is a rectangle.
- 4. If AB = CD, BC = DA;  $AC \neq BD$ , then ABCD is a parallelogram.
- > Diagonals of a square, rhombus, rectangle and parallelogram always bisect each other.
- > Diagonals of rhombus and square bisect each other at right angle.
- > All given points are collinear, if the area of the obtained polygon is zero.
- > Three given points are collinear, if the area of triangle is zero.
- > Centroid is the point of intersection of the three medians of a triangle. In the figure, G is the centroid of a triangle ABC.



- > Centroid divides each median of a triangle in a ratio of 2 : 1 from vertex to base of the side.
- For  $x \neq y$ , then  $(x, y) \neq (y, x)$  and if (x, y) = (y, x), then x = y.
- To plot a point P(3, 4) in the cartesian plane.
  - (i) A distance of 3 units along X-axis.

(ii) A distance of 4 units along Y-axis.



> The distance between two points *i.e.*,  $P(x_1, y_1)$  and  $Q(x_2, y_2)$  is

$$d = \left| \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \right|^2$$

> The distance of a point P(x, y) from origin is  $\sqrt{x^2 + y^2}$ 

Co-ordinates of point (x, y) which divides the line segment by joining the points ( $x_1$ ,  $y_1$ ) and ( $x_2$ ,  $y_2$ ) in the ratio m : n internally are

$$x = \left(\frac{mx_2 + nx_1}{m+n}\right)$$
 and  $y = \left(\frac{my_2 + ny_1}{m+n}\right)$ 

> Co-ordinates of mid-point of the line segment by joining the points  $(x_1, y_1)$  and  $(x_2, y_2)$  are

$$x = \left(\frac{x_2 + x_1}{2}\right)$$
 and  $y = \left(\frac{y_2 + y_1}{2}\right)$ 

**G** Know the Facts

- Co-ordinate geometry is the system of geometry where the position of points on the plane is described using an ordered pair of numbers.
- > Cartesian plane was discovered by *Rene Descartes*.
- > The other name of co-ordinate geometry is Analytical Geometry.
- > Co-ordinate Geometry acts as a bridge between the Algebra and Geometry.
- > Medians of a triangle are concurrent. The point of concurrency is called the centroid.
- > Trisection of a line segment means dividing it into 3 equal parts, so 2 points are required.
- > Centroid of a triangle divides its median in the ratio of 2 : 1.

# How is it done on the GREENBOARD?

Q.1. Find the point of trisection of the line segment joining the points (5, -6) and (-7, 5).

Solution

Step	I: Diagrammatic	repres	entation.
А	$(x_{1'}y_1)$	$(x_2, y_2)$	В
(5 -6)	ן ק	0	(-7.5)

Points of trisections divides line segment into three equal parts.

i.e., *AP* = *PQ* = *QB*  **Step II**: The ratio of AP : PQ : *QB* = 1 : 1 : 1

or,

$$\frac{AP}{PB} = \frac{1}{2}$$
 and  $\frac{AQ}{QB} = \frac{2}{1}$ 

Step III: finding co-ordinates of P using section formula.

$$\begin{array}{cccc}
A & P & B \\
(5, -6) & (x_1, y_1) & (-7, 5) \\
& 1:2
\end{array}$$

$$Y_1 = \frac{1(5) + 2(-6)}{1+2} = \frac{-7}{3}$$

$$\therefore P(x_1, y_1) \text{ is } \left(1, \frac{-7}{3}\right).$$

Step IV: Finding co-ordinates of Q using section formula

$$\begin{array}{c} A \\ (5,-6) \\ (x_2, y_2) \\ (x_2, y_2) \\ (-7,5) \\ 2:1 \\ x_2 = \frac{2(-7) + 1(5)}{2+1} = \frac{-9}{3} = -3 \\ y_2 = \frac{2(5) + 1(-6)}{2+1} = \frac{4}{3} \\ \therefore Q(x_2, y_2) \text{ is } \left(-3, \frac{4}{3}\right). \end{array}$$

## Very Short Answer Type Questions

## **AI** Q. 1. In which quadrant lies the point which divides **AI** Q. 3. If t

the line segment joining the points (8, – 9) and (2, 3) in ratio 1 : 2 internally ?

**U** [CBSE SQP, 2020]

Sol. IV quadrant.

[CBSE SQP Marking Scheme, 2020]

### **Detailed Solution:**

$$\frac{1}{A(8'-9)} \frac{2}{P(x'y)} \frac{2}{B(2,3)}$$
  
 $m = 1, n = 2$   
Given,  $(x_1, y_1) = (8, -9)$   
 $(x_2, y_2) = (2, 3)$   
 $(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right] \frac{1}{2}$   
 $(x, y) = \left[\frac{1 \times 2 + 2 \times 8}{1+2}, \frac{1 \times 3 + 2 \times (-9)}{1+2}\right]$   
 $(x, y) = \left[\frac{2 + 16}{3}, \frac{3 - 18}{3}\right]$   
 $(x, y) = \left[\frac{18}{3}, \frac{-15}{3}\right]$   
 $(x, y) = (6, -5)$ 

Hence, the point (6, -5) lies in IV quadrant.  $\frac{1}{2}$ 

### **AI** Q. 2. Find the distance between the points

 $(a\cos\theta + b\sin\theta, 0)$  and  $(0, a\sin\theta - b\cos\theta)$ .

**Sol.** Here, 
$$x_1 = a \cos \theta + b \sin \theta$$
,  $y_1 = 0$   
and  $x_2 = 0$ ,  $y_2 = a \sin \theta - b \cos \theta$ 

:. Distance = 
$$\left| \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \right| \frac{1}{2}$$

$$= \left| \sqrt{(0 - a\cos\theta - b\sin\theta)^2 + (a\sin\theta - b\cos\theta - 0)^2} \right|$$
$$= \left| \sqrt{(-1)^2 (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2} \right|$$
$$= \left| \sqrt{a^2 \cos^2\theta + b^2 \sin^2\theta + 2ab\cos\theta \sin\theta} \right|$$
$$= \left| \sqrt{a^2 (\sin^2\theta + b^2 \cos^2\theta - 2ab\sin\theta \cos\theta)} \right|$$
$$= \left| \sqrt{a^2 (\sin^2\theta + \cos^2\theta) + b^2 (\sin^2\theta + \cos^2\theta)} \right|$$
$$= \left| \sqrt{a^2 \times 1 + b^2 \times 1} \right|$$
$$= \left| \sqrt{a^2 + b^2} \right| \text{ unit }$$

Q. 3. If the point P(k, 0) divides the line segment joining the points A(2, -2) and B(-7, 4) in the ratio 1 : 2, then find the value of k.

Sol.  

$$A(2,-2)$$

$$k = \frac{1(-7)+2(2)}{1+2}$$

$$k = \frac{1(-7)+2(2)}{1+2}$$

$$\begin{bmatrix} \because x = \frac{mx_2 + nx_1}{m+n} \end{bmatrix}$$

$$k = \frac{-7+4}{3}$$

$$3k = -3$$

$$k = -1.$$

$$1/2$$

**A** Q. 4. The point P on X-axis equidistant from the points A(-1, 0) and B(5, 0).

### A [CBSE OD Set-I, 2020]

**Sol.** Let the position of the point P on X-axis be (x, 0), then

$$X \stackrel{A(-1,0)}{\longleftarrow} X \stackrel{A(-1,0)}{\longleftarrow} X$$

$$PA^{2} = PB^{2}$$

$$\Rightarrow (x + 1)^{2} + (0)^{2} = (5 - x)^{2} + (0)^{2}$$

$$\Rightarrow x^{2} + 2x + 1 = 25 + x^{2} - 10x$$

$$\Rightarrow 2x + 10x = 25 - 1$$

$$\Rightarrow 12x = 24$$

$$\Rightarrow x = 2$$
Hence, the point P (x, 0) is (2, 0).

Q. 5. Find the co-ordinates of the point which is reflection of point (-3, 5) in X-axis.

### A [CBSE OD Set-I, 2020]

**Sol.** By using the graph of coordinate plane, we have the reflection of point (-3, 5) is *x*-axis is (-3, -5). <sup>1</sup>/<sub>2</sub>





### 1 mark each

Sol. Here, 
$$x_1 = 6, y_1 = 5$$
  
 $A(6,5)$   $3$   $1$   $B(4, y)$   
and  $x_2 = 4, y_2 = y$   
Then  $x = \frac{mx_2 + nx_1}{m+n}$  and  $y = \frac{my_2 + ny_1}{m+n}$   $\frac{1}{2}$   
 $\therefore$   $2 = \frac{3 \times y + 1 \times 5}{3+1} = \frac{3y+5}{4}$   
 $\Rightarrow$   $3y + 5 = 8$   
 $\Rightarrow$   $3y = 8 - 5 = 3$   
 $\Rightarrow$   $y = 1$ .  $\frac{1}{2}$   
Events the point (1, 4).  $A$  [CBSE Delhi Set-I, II, III, 2019]  
Sol. Let the point A be  $(x, y)$   
 $\therefore$   $\frac{x+1}{2} = 2$  and  $\frac{4+y}{2} = -3$   $\frac{1}{2}$   
 $\Rightarrow$   $x = 3$  and  $y = -10$   $\frac{1}{2}$   
 $\therefore$  Point A is  $(3, -10)$   
[CBSE Marking Scheme, 2019]

Q. 8. The distance between point A(5, -3) and B(13, m) is 10 units. Calculate the value of m.

[CBSE Delhi Board term, 2019] en Li **Topper Answer, 2019** A = 5, -3B = 13, mSol. AB = 1 busits formula; distance (13-5)2+ (m+3) 10 On squaring,  $8^2 + (m+3)^2$ = 100  $(m+3)^2 = 100 - 64$  $\sqrt{(m+3)^2} = \sqrt{36}$  $(m+3)^2 = \frac{1}{26}$ 2> -7 Considering only positive palue; m 26-3 Am=3

1

Q. 9. Find the value of *a*, for which point P  $\left(\frac{a}{3}, 2\right)$  is the

midpoint of the line segment joining the points Q(-5, 4) and R(-1, 0). A [CBSE SQP 2018]

Sol. 
$$\left(\frac{-5+(-1)}{2}, \frac{4+0}{2}\right) = \left(\frac{a}{3}, 2\right)$$
$$\frac{a}{3} = \frac{-6}{2} \Rightarrow a = -9$$
1

[CBSE Marking Scheme, 2018]

 $\frac{1}{2}$ 

 $\frac{1}{2}$ 

*:*..

**Detailed Solution:** 

$$\begin{array}{c} Q & P \\ (-5, 4) & \left(\frac{a}{3}, 2\right) & (-1, 0) \end{array}$$

P is the mid-point of *QR* 

or, 
$$\frac{a}{3} = \frac{-5 + (-1)}{2}$$
  
or,  $\frac{a}{3} = \frac{-6}{2}$   
or,  $a = -9$ .

Q. 10. A(5, 1), B(1, 5) and C(-3, -1) are the vertices of  $\triangle$ ABC. Find the length of median AD.



## **Short Answer Type Questions-I**

**AI** Q. 1. Find the point on X-axis which is equidistant from the points (2, – 2) and (– 4, 2).

**U** [CBSE SQP, 2020-21]

**Sol.** Let P(x, 0) be a point on *x*-axis

$$PA = PB \qquad \frac{1}{2}$$
$$PA^2 = PB^2 \qquad \frac{1}{2}$$

$$(x-2)^{2} + (0+2)^{2} = (x+4)^{2} + (0-2)^{2}$$
$$x^{2} + 4 - 4x + 4 = x^{2} + 16 + 8x + 4$$
$$-4x + 4 - 8x + 16$$

$$-4x + 4 = 8x + 16$$
 <sup>1</sup>/<sub>2</sub>

$$x = -1$$
 <sup>1</sup>/<sub>2</sub>

Hence co-ordinate of required point are (-1, 0).

$$AD = \sqrt{(5+1)^2 + (1-2)^2} = \sqrt{37}$$
 unit <sup>1</sup>/<sub>2</sub>

[CBSE Marking Scheme, 2018]

Detailed Solution:



Here D is the mid-point of BC.

Then, the coordinates of D

$$= \left(\frac{1-3}{2}, \frac{5-1}{2}\right) = (-1, 2) \quad \frac{1}{2}$$
$$AD = \left|\sqrt{(5+1)^2 + (1-2)^2}\right|$$
$$= \left|\sqrt{36+1}\right| = \left|\sqrt{37}\right|$$

Hence, the length of AD is  $\sqrt{37}$  unit.

Q. 11. If the distance between the points (4, *k*) and (1, 0) is 5, then what can be the possible values of *k* ?

Sol. Using distance formula,

$$\left| \sqrt{(4-1)^2 + (k-0)^2} \right| = 5$$
<sup>1/2</sup>

or,  $3^2 + k^2 = 25$  $k = \pm 4$  <sup>1</sup>/<sub>2</sub>

[CBSE Marking Scheme, 2017]

 $\frac{1}{2}$ 

### 2 marks each

**All** Q. 2. P(-2, 5) and Q(3, 2) are two points. Find the coordinates of the point R on PQ such that PR = 2QR.

### A [CBSE SQP, 2020-21]

Sol. 
$$PR: QR = 2:1$$
  
 $R(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$   
 $R\left(\frac{1(-2) + 2(3)}{2+1}, \frac{1(5) + 2(2)}{2+1}\right)$   
 $R\left(\frac{4}{2}, 3\right)$ 

$$\left(\frac{4}{3},3\right)$$
.  $\frac{1}{2}$ 

[CBSE Marking Scheme, 2020-21]



Q. 3. In parallelogram ABCD, A(3, 1), B(5, 1) C(*a*, *b*) and D(4, 3) are the vertices. Find vertex C(*a*, *b*).

A [CBSE Board Term, 2019]



**Sol.** Let 
$$x = \frac{6-6}{5} = 0$$

5 1 [CBSE Marking Scheme, 2018]

 $y = \frac{-10+15}{5} = 1$ 

**Detailed Solution:** 

Δ	P(x, y)	P
(-2,5)	2:3	(3,-5)
Since,	$x = \frac{mx_2 + nx_1}{m + n}$	
and	$y = \frac{my_2 + ny_1}{m+n}$	

$$x = \frac{2 \times 3 + 3 \times (-2)}{2 + 3} = 0 \qquad \frac{1}{2}$$

 $y = \frac{2 \times (-5) + 3 \times 5}{2 + 3} = \frac{5}{5} = 1 \frac{1}{2}$ 

[CBSE Marking Scheme, 2018]

and

1

So, the point is (0, 1).

Q. 5. Find the ratio in which P(4, *m*) divides the line segment joining the points A(2, 3) and B(6, − 3). Hence find *m*. C + A [CBSE Delhi/OD Set-2018]

Sol. A 
$$\begin{pmatrix} k & 1 \\ (2,3) & P(4,m) & (6,-3) \end{pmatrix}$$
  
Let  $AP:PB = k:1$   
 $\frac{6k+2}{k+1} = 4$   
 $\Rightarrow$   $k = 1$ , ratio is  $1:1$  1  
Hence  $m = \frac{-3+3}{2} = 0$   $\frac{1}{2}$ 

**Detailed Solution:** 

## **Topper Answer, 2018**

 $\frac{1}{2}$ 

 $\frac{1}{2}$ 

	Points $A(2,3)$ , $G(6,-3)$ divided by $P(-1,m)$ .
6	R. au Orching Armuda
E	$P(4,m) = \left(\frac{mx_{a} + nx_{i}}{L_{a} + m}\right)$
_	$(4, m) = (\frac{k+2}{k+1}, -\frac{2k+3}{k+1})$
	=) $\frac{6k_{12}}{k_{11}} = 4$
	Ckrz=4k+4 2k=2 The ratio is life k=1.
2	Now, M3 -3k+3 k+1
	$m = -\frac{3+3}{141}$

**All** Q. 6. If  $\left(1, \frac{p}{3}\right)$  is the mid point of the line segment joining the points (2, 0) and  $\left(0, \frac{2}{9}\right)$ , then show that the line 5x + 3y + 2 = 0 passes through the point (-1, 3*p*).

A [CBSE SQP, 2017]

**Sol.** Since  $\left(1, \frac{p}{3}\right)$  is the mid point of the line segment

2

joining the points (2, 0) and 
$$\left(0, \frac{2}{9}\right)$$
.  

$$\therefore \qquad \frac{p}{3} = \frac{0 + \frac{2}{9}}{2} = \frac{2p}{3} = \frac{2}{9}$$

$$p = \frac{1}{3} \qquad 1$$

Hence, the line 5x + 3y + 2 = 0, passes through the point (-1, 1) as 5(-1) + 3(1) + 2 = 0. **1** [CBSE Marking Scheme, 2017]

Q. 7. In what ratio does the point P(- 4, 6) divide the line segment joining the points A(- 6, 10) and B(3, - 8)? A [Delhi Comptt. Set-I, II, III 2017]

Sol. Let	AP: PB = k: 1	1/2
÷	$\frac{3k-6}{k+1} = -4$	
or,	3k-6 = -4k-4	1
or,	7k = 2	
	$k = \frac{2}{7}$	
Hence,	AP: PB = 2:7	1/2
	[CBSE Marking Sche	me, 2017]

Q. 8. If the line segment joining the points A(2, 1) and B(5, -8) is trisected at the points P and Q, find the coordinates P.

	A [Outside I	Jeini Com	ptt. Set-1, 111, 2	.017]
<b>Sol.</b> A (2, 1)	P	Q	——————————————————————————————————————	
Let P(x,	y) divides AB	in the ratio	01:2.	1
.:. Using	section form	ıla		
	<i>x</i> =	$= \frac{1 \times 5 + 2}{1 + 2}$	$\frac{\times 2}{2} = 3$	
and	<i>y</i> :	$=\frac{1\times-8+}{1+2}$	$\frac{2\times 1}{2} = -2$	1
Hence co	oordinates of	P are (3. –	2).	

Q. 9. If the distances of P(x, y) from A(5, 1) and B(-1, 5) are equal, then prove that 3x = 2y.





Q. 10. A line intersects the Y-axis and X-axis at the points P and Q respectively. If (2, – 5) is the mid-point of PQ, then find the coordinates of P and Q.



Q. 11. The *x*-coordinate of a point P is twice its *y*-coordinate. If P is equidistant from Q(2, -5) and R(-3, 6), find the co-ordinates of P.

DO

U [CBSE, Delhi Set I, II, III, 2016]

1

 $\frac{1}{2}$ 

**Sol.** Let the point P be (2y, y)

0.

or, 
$$\left|\sqrt{(2y-2)^2 + (y+5)^2}\right| = \left|\sqrt{(2y+3)^2 + (y-6)^2}\right|^{\frac{1}{2}}$$

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Solving to get y = 8

Hence, coordinates of point P are (16, 8). <sup>1</sup>/<sub>2</sub> [CBSE Marking Scheme, 2016]

Detailed Solution:

Let the point P be (2y, y)

Given, 
$$PQ = PR$$
 **1**  
or,  $\left|\sqrt{(2y-2)^2 + (y+5)^2}\right| = \left|\sqrt{(2y+3)^2 + (y-6)^2}\right|_{\frac{1}{2}}$ 

Squaring both sides

$$4y^{2} - 8y + 4 + y^{2} + 25 + 10y$$
  
= 4y<sup>2</sup> + 12y + 9 + y<sup>2</sup> + 36 - 12y  
or, 2y = 16 or y = 8

Hence, coordinates of point P are (16, 8).

Q. 12. Find the ratio in which Y-axis divides the line segment joining the points A(5, -6) and B(-1, -4). Also find the co-ordinates of the point of division.
A [CBSE, Delhi Set I, II, III, 2016]

**Sol.** Let the point on *y*-axis be (0, y) and AP : PB = k : 1.  $\frac{1}{2}$ 

Therefore 
$$\frac{5-k}{k+1} = 0$$
 gives  $k = 5$ 

Hence, required ratio is 5:1  $\frac{1}{2}$ 

$$y = \frac{-4(5)-6}{6} = \frac{-13}{3} \qquad \frac{1}{2}$$

Hence, point on Y-axis is 
$$\left(0, \frac{-13}{3}\right)$$
.

### [CBSE Marking Scheme, 2016]

**AI** Q. 13. If the point P(x, y) is equidistant from the points Q(a + b, b - a) and R(a - b, a + b), then prove that bx = ay. **(OD, Set I, II, III, 2016) (A)** [CBSE SQP, 2016]

Sol. 
$$|PQ| = |PR|$$
  
 $\left|\sqrt{[x - (a + b)]^2 + [y - (b - a)]^2}\right|$   
 $= \left|\sqrt{[x - (a - b)]^2 + [y - (b + a)]^2}\right|$ 



Q. 14. Prove that the point (3, 0), (6, 4) and (-1, 3) are the vertices of a right angled isosceles triangle.

U [CBSE, OD Set I, II, III, 2016]

$$\therefore AB^2 = (3-6)^2 + (0-4)^2 = 9 + 16 = 25$$
 <sup>1</sup>/<sub>2</sub>

$$BC^{2} = (6+1)^{2} + (4-3)^{2} = 49 + 1 = 50$$
<sup>1/2</sup>

and 
$$CA^2 = (-1 - 3)^2 + (3 - 0)^2 = 16 + 9 = 25$$
 <sup>1</sup>/<sub>2</sub>

$$AB^2 = CA^2$$
 or,  $AB = CA$ 

∴ Triangle is isosceles.

$$25 + 25 = 50$$
Also,  $AB^2 + CA^2 = BC^2$  <sup>1</sup>/<sub>2</sub>
  
C (-1, 3)
  
A(3, 0)
  
B (6, 4)

Since, Pythagoras theorem is verified, therefore triangle is a right angled triangle.

### [CBSE Marking Scheme, 2016]

Q. 15. If the mid-point of the line segment joining  

$$A\left[\frac{x}{2}, \frac{y+1}{2}\right]$$
 and  $B(x + 1, y - 3)$  is C(5, - 2), find  
 $x, y.$   $\bigcup$  [CBSE Board Term-2, 2016]

Sol.  
At mid-point of 
$$AB = \left(\frac{\frac{x}{2} + x + 1}{2}\right) = 5$$
  
or,  
 $x = 6$  1  
 $\left(\frac{\frac{y+1}{2} + y - 3}{2}\right) = -2$   
 $y + 1 + 2y - 6 = -8$   
or,  
 $y = -1$ .

### [CBSE Marking Scheme, 2016]

Sol

Q. 16. If A (5, 2), B (2, – 2) and C (–2, *t*) are the vertices of a right angled triangle with  $\angle B = 90^\circ$ , then find the value of *t*. U [CBSE Board, 2015]

Sol. 
$$AB^2 = (2-5)^2 + (-2-2)^2 = 9 + 16 = 25$$
  
 $BC^2 = (-2-2)^2 + (t+2)^2 = 16 + (t+2)^2$   
 $AC^2 = (5+2)^2 + (2-t)^2 = 49 + (2-t)^2$   
A (5, 2)  
B (2, -2)  
C (-2, t)  
Since,  $\triangle ABC$  is a right angled triangle.

 $AC^2 = AB^2 + BC^2$ 

or, 
$$49 + (2-t)^2 = 25 + 16 + (t+2)^2$$
  
or,  $49 + 4 - 4t + t^2 = 41 + t^2 + 4t + 4$   
or,  $53 - 4t = 45 + 4t$   
or,  $8t = 8$   
 $\therefore$   $t = 1$  1

[CBSE Marking Scheme, 2015]

Q. 17. Show that the points (a, a), (-a, -a) and  $(-\sqrt{3}a, \sqrt{3}a)$ are the vertices of an equilateral triangle.

U [Foreign Set I, II, III, 2015]

Let A(a, a), B(-a, -a) and C (
$$-\sqrt{3}a, \sqrt{3}a$$
)  
Here,  $AB = \left|\sqrt{(a+a)^2 + (a+a)^2}\right| = 2\sqrt{2}a$  unit  $\frac{1}{2}$   
 $BC = \left|\sqrt{(-a+\sqrt{3}a)^2 + (-a-\sqrt{3}a)^2}\right|$   
 $= \left|\sqrt{a^2 + 3a^2 - 2\sqrt{3}a^2 + a^2 + 2\sqrt{3}a^2 + 3a^2}\right|$ 

$$= \sqrt{8a^2} = 2\sqrt{2}a \text{ unit} \qquad \frac{1}{2}$$

and 
$$AC = \left| \sqrt{(a + \sqrt{3}a)^2 + (a - \sqrt{3}a)^2} \right|^{\frac{1}{2}}$$

$$\left|\sqrt{a^2 + 3a^2 + 2\sqrt{3}a^2 + a^2 + 3a^2 - 2\sqrt{3}a^2}\right| = 8a^2$$

 $= 2\sqrt{2}a$  unit

Since AB = BC = AC, therefore ABC is an equilateral  $\frac{1}{2}$ triangle.

### [CBSE Marking Scheme, 2015]

3 marks each

## Short Answer Type Questions-II

**AI** Q. 1. If the point C(– 1, 2) divides internally the line segment joining A(2, 5) and B(x, y) in the ratio 3 : 4, find the coordinates of B.

A(2, 5) C(-1, 2) B(x, y) 
$$\frac{1}{2}$$

$$-1 = \frac{mx_2 + nx_1}{m+n}$$

$$= \frac{3 \times x + 4 \times 2}{3 + 4} = \frac{3x + 8}{7} \quad \frac{1}{2}$$

$$\Rightarrow \qquad 3x = -15$$
  
$$\Rightarrow \qquad x = -5 \qquad \frac{1}{2}$$
  
and 
$$2 = \frac{my_2 + ny_1}{2}$$

m + n

$$= \frac{3 \times y + 4 \times 5}{3 + 4} = \frac{3y + 20}{7} \frac{1}{2}$$

3y + 20 = 14

and

 $\Rightarrow$  $\Rightarrow$ 

 $\Rightarrow$ 

$$3y = 14 - 20 = -6$$
  
 $y = -2$ 

Hence, the coordinates of B(x, y) is (-5, -2).  $\frac{1}{2}$ 

 $\Rightarrow$ 

3x + 8 = -7

AI Q. 2. If the mid-point of the line segment joining the points A(3, 4) and B(k, 6) is P(x, y) and x + y − 10 = 0, find the value of k. A+U [CBSE, OD Set-I, 2020]
Sol. Since, the mid point of A and B is P.

**ol.** Since, the mid point of A and B is P,  
A(3, 4) 
$$P(x, y)$$
  $B(k, 6)$ 

 $\frac{3+k}{2} = x$ 

and

Also, given, x + y - 10 = 0

Substituting the value of  $x = \frac{3+k}{2}$  and y = 5, we

 $y = \frac{4+6}{2} = \frac{10}{2} = 5$ 

get

$$\Rightarrow \frac{3+k}{2} + 5 - 10 = 0 \qquad 1$$

$$\Rightarrow \frac{3+k}{2} = 5$$

$$\Rightarrow \qquad 3+k = 10$$

$$\Rightarrow \qquad k = 10 - 3 = 7.$$
Hence, the value of k is 7.

**All** Q. 3. The line segment joining the points A(2, 1) and B(5, -8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by 2x - y + k = 0, find the value of k.

A [CBSE Delhi Set-I, II, III, 2019]

Sol. 
$$AP: PB = 1:2$$
  $\frac{1}{2}$   
 $AP: PB = 1:2$   $\frac{1}{2}$   
 $A(2, 1)$   $P(x, y)$   $B(5, -8)$   $\frac{1}{2}$   
 $x = \frac{4+5}{3} = 3 \text{ and } y = \frac{2-8}{3} = -2$   $\frac{1}{2} + \frac{1}{2}$   
Thus point P is  $(3, -2)$ .  
Point  $(3, -2)$  lies on  $2x - y + k = 0$   
 $\Rightarrow$   $6 + 2 + k = 0$   
 $\Rightarrow$   $k = -8$ . 1  
[CBSE Marking Scheme, 2019]

**Detailed Solution:** 



The given points are A (2, 1) and B (5, –8). Given, P and Q trisects the line segment AB

AP = PQ = QB PB = PQ + QB = AP + AP = 2APAP : PB = AP : 2AP = 1 : 2

 $\therefore$  P divides the line segment AB in the ratio 1 : 2. We know that, coordinates of the point dividing the

line segment joining the points  $(x_1, y_1)$  and  $(x_2, y_2)$  in (mr + mr - mr + mr)

the ratio m: n is given by  $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$ .

: Coordinates of P

*:*..

1

$$= \left(\frac{1 \times 5 + 2 \times 2}{1 + 2}, \frac{1 \times (-8) + 2 \times 1}{1 + 2}\right)$$
$$= \left(\frac{5 + 4}{3}, \frac{-8 + 2}{3}\right)$$
$$= \left(\frac{9}{3}, \frac{-6}{3}\right) = (3, -2)$$

Point P(3, -2) lies on the given line 2x - y + k = 0

 $\therefore \qquad 2 \times 3 - (-2) + k = 0$   $\Rightarrow \qquad 6 + 2 + k = 0$   $\Rightarrow \qquad 8 + k = 0$   $\Rightarrow \qquad k = -8$ Thus, the value of k is -8

Thus, the value of k is -8.

### **COMMONLY MADE ERROR**

Some Candidates use mid point formula instead of section formula to find the coordinates of P. Some candidates also make calculation errors.

### **ANSWERING TIP**

- Understand the concept if a point divides a line segment in the ratio  $m : n (m \neq n)$ then m : n is not equal to n : m.
- Q. 4. Find the ratio in which the line x 3y = 0 divides the line segment joining the points (-2, - 5) and (6, 3). Find the coordinates of the point of intersection. A [CBSE OD Set-I, II, III, 2019]

**Sol.** Let the line 
$$x - 3y = 0$$
 intersect the segment

A 
$$\frac{k:1}{(-2,-5)}$$
 P (6, 3)  
 $x - 3y = 0$ 

joining A(-2, -5) and B(6, 3) in the ratio 
$$k : 1$$
  $\frac{1}{2}$ 

$$\therefore \text{ Coordinates of P are } \left(\frac{6k-2}{k+1}, \frac{3k-5}{k+1}\right) \qquad 1$$

P lies on 
$$x - 3y = 0 \Rightarrow \frac{6k - 2}{k + 1} = 3\left(\frac{3k - 5}{k + 1}\right)$$
  
$$\Rightarrow \qquad k = \frac{13}{3}$$

$$\therefore \text{ Ratio is } 13:3 \qquad 1$$
  

$$\Rightarrow \text{ Coordinates of P are } \left(\frac{9}{2}, \frac{3}{2}\right) \qquad \frac{1}{2}$$

### [CBSE Marking Scheme, 2019]

### **Detailed Solution:**

Let the ratio in which line x - 3y = 0 divides the line segment is k:1



Using section formula, we get

$$x = \frac{k \times 6 + 1 \times (-2)}{k+1}$$
$$= \frac{6k-2}{k+1} \dots (i) \frac{1}{2}$$
and
$$y = \frac{k \times 3 + 1 \times (-5)}{k+1}$$
$$= \frac{3k-5}{k+1} \dots (ii) \frac{1}{2}$$

The point P(x, y) lies on the line, hence satisfies the equation of the given line.

k + 1

$\Rightarrow$	$\frac{6k-2}{k+1} - 3\left(\frac{3k-5}{k+1}\right) = 0$
$\Rightarrow$	6k - 2 - 3(3k - 5) = 0
$\Rightarrow$	6k - 2 - 9k + 15 = 0
$\Rightarrow$	-3k + 13 = 0
$\Rightarrow$	$k = \frac{13}{3}$

Hence, the required ratio is 13:3

Now, substituting value of *k* in *x* and *y*, we get

$$x = \frac{6 \times \left(\frac{13}{3}\right) - 2}{\frac{13}{3} + 1}$$
$$= \frac{78 - 6}{16}$$
$$= \frac{72}{16}$$
$$= \frac{9}{2}$$

and

$$y = \frac{3(3)^{-3}}{\frac{13}{3}+1} = \frac{8 \times 3}{16}$$

 $3(\frac{13}{13}) - 5$ 

 $\frac{1}{2}$ 

Hence, the coordinates of point of intersection

$$P(x, y) = \left(\frac{9}{2}, \frac{3}{2}\right)$$
 <sup>1</sup>/<sub>2</sub>

 $\frac{21}{16} = \frac{3}{2}$ 

Q. 5. If A(- 2, 1), B(a, 0), C(4, b) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of *a* and *b*. Hence find the lengths of its sides.

A [CBSE Delhi/OD, 2018]



Therefore mid point P of BD is same as mid point of AC  $\frac{1}{2}$ 

$$\left(\frac{a+1}{2}, \frac{2}{2}\right) = \left(\frac{-2+4}{2}, \frac{1+b}{2}\right)$$
  
 $\frac{a+1}{2} = 1 \text{ and } \frac{b+1}{2} = 1$ 

 $\Rightarrow a = 1, b = 1$ . Therefore, length of sides are  $\sqrt{10}$ units each.  $\frac{1}{2} + 1$ [CBSE Marking Scheme, 2018]

### **Detailed Solution:**

.

 $\frac{1}{2}$ 

 $\frac{1}{2}$ 

 $\Rightarrow$ 

We know that diagonals of parallelogram bisect each other.

.:. Mid-point of diagonal AC

$$\left(\frac{-2+4}{2},\frac{1+b}{2}\right) = \left(1,\frac{1+b}{2}\right)$$

Mid-point of diagonal BD

$$\left(\frac{a+1}{2}, \frac{0+2}{2}\right) = \left(\frac{a+1}{2}, 1\right)$$
 <sup>1/2</sup>

Mid point of diagonal AC = mid point of diagonal BD

$$1 = \frac{a+1}{2} \text{ and } \frac{1+b}{2} = 1$$

$$2 = a+1 \text{ and } 1+b=2$$

$$\therefore \qquad a = 1 \text{ and } b = 1 \frac{1}{2}$$
Again,
$$AB = \left| \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \right| \frac{1}{2}$$

$$= \left| \sqrt{(1+2)^{2} + (0-1)^{2}} \right|$$

$$AB = \left| \sqrt{9+1} \right| = \sqrt{10} \text{ unit}$$
and
$$BC = \left| \sqrt{(x_{2} - x_{1})^{2} + (y_{2} - y_{1})^{2}} \right|$$

$$= \left| \sqrt{(4-1)^{2} + (1-0)^{2}} \right|$$

$$BC = \left| \sqrt{9+1} \right|$$

$$= \sqrt{10} \text{ unit}$$

ABCD is a parallelogram (Given)

 $\therefore \qquad AB = CD = \sqrt{10} \text{ unit}$ 

and 
$$BC = AD = \sqrt{10}$$
 unit

Q. 6. The points A(1, - 2), B(2, 3), C(k, 2) and D(-4, -3) are the vertices of a parallelogram. Find the value of k.  $\bigcirc$  +  $\bigcirc$  [CBSE SQP, 2018]



$$\Rightarrow \quad \left(\frac{1+k}{2}, \frac{-2+2}{2}\right) = \left(\frac{-4+2}{2}, \frac{-3+3}{2}\right)$$

 $\frac{1+k}{2} = \frac{-2}{2}$ 

 $\Rightarrow$ 

 $\Rightarrow$ 

k = -3

[CBSE Marking Scheme, 2018]

### **COMMONLY MADE ERROR**

 Mostly candidates do not use the mid point formula. Generally they use distance formula.

### ANSWERING TIP

- Candidates should use mid point formula by which they can get correct solution in lesser time.
- Q. 7. If coordinates of two adjacent vertices of a parallelogram are (3, 2), (1, 0) and diagonals bisect each other at (2, 5), find coordinates of the other two vertices. U [CBSE Comptt. Set I, II, III, 2018]
- **Sol.** Let the coordinates of C and D be (*a*, *b*) and (*c*, *d*).



$$\frac{3+a}{2} = 2 \Longrightarrow a = 1 \qquad \frac{1}{2}$$

and 
$$\frac{2+b}{2} = -5 \Rightarrow b = -12$$
 <sup>1</sup>/<sub>2</sub>

and 
$$\frac{c+1}{2} = 2 \Rightarrow c = 3$$
  $\frac{1}{2}$ 

ad 
$$\frac{d+0}{2} = -5 \Rightarrow d = -10 \qquad \frac{1}{2}$$

Coordinates of C and D are (1, – 12) and (3, – 10) 1 [CBSE Marking Scheme, 2018]

Q. 8. In what ratio does the point  $\left(\frac{24}{11}, y\right)$  divide the line segment joining the points P (2, – 2) and Q (3, 7)? Also find the value of y.

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24m+24n= 33m+22n 9 m 21 = m n . The given point dinder the line segment In ratio 2:9. m=2 and m= 9, Taking 7m -2n (from ()) 42 m+n 7(2)-2(9) 4 2+9 14-18. 4 11 4 X

Q. 9. Find the co-ordinates of the points which divide the line segment joining the points (5, 7) and (8, 10) in 3 equal parts. [Board SQP, 2016] [A] [CBSE OD Comptt. Set-II, 2017]

Sol. A 
$$\leftarrow$$
 B  
 $(5,7)$  P Q  $(8,10)$   
Let P(x, y) and Q(x<sub>1</sub>, y<sub>1</sub>) trisect AB.  
 $\therefore$  P divides AB in the ratio 1 : 2  
 $\therefore$   $x = \frac{1(8) + 2(5)}{3} = 6$ , 1  
and  $y = \frac{1(10) + 2(7)}{3} = 8$   
 $\therefore$  P(6, 8)  
And Q is the mid point of PB. 1  
 $x_1 = \frac{6+8}{2} = 7$   
 $y_1 = \frac{8+10}{2} = 9$   
 $\therefore$  Q(7, 9) 1  
[CBSE Marking Scheme, 2017]

Q. 10. Show that  $\triangle ABC$  with vertices A(- 2, 0), B(0, 2) and C(2, 0) is similar to  $\triangle DEF$  with vertices D(-4, 0), E(0, 4) and F(4, 0).

> A [Board Foreign Set-I, II 2017; CBSE Delhi Set-I, II, III, 2017]

Sol. Using distance formula

$$AB = \left| \sqrt{(0+2)^2 + (2-0)^2} \right| = \left| \sqrt{4+4} \right|$$
  
=  $2\sqrt{2}$  units  
$$BC = \left| \sqrt{(2-0)^2 + (0-2)^2} \right| = \left| \sqrt{4+4} \right|$$
  
=  $2\sqrt{2}$  units  $\frac{1}{2}$   
$$CA = \left| \sqrt{(-2-2)^2 + (0-0)^2} \right| = \left| \sqrt{16} \right|$$
  
= 4 units

$$DE = \left| \sqrt{(0+4)^2 + (4-0)^2} \right| = \left| \sqrt{32} \right|$$
  
=  $4\sqrt{2}$  units <sup>1/2</sup>  
$$EF = \left| \sqrt{(4-0)^2 + (0-4)^2} \right| = \left| \sqrt{32} \right|$$
  
=  $4\sqrt{2}$  units  
$$FD = \left| \sqrt{(-4-4)^2 + (0-0)^2} \right| = \left| \sqrt{64} \right|$$
  
= 8 units <sup>1/2</sup>

and

Since, ratio of the corresponding sides of two similar  $\Delta s$  is equal.

*i.e.*, 
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$
 <sup>1</sup>/<sub>2</sub>

or, 
$$\frac{2\sqrt{2}}{4\sqrt{2}} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{4}{8} = \frac{1}{2}$$
 1

$$\therefore \quad \Delta ABC \sim \Delta DEF \qquad \text{Hence Proved.}$$

[Foreign Set-III, 2015]

3



**Sol.** The co-ordinates of B are (5, 0)Let co-ordinates of C be (x, y) (AB = 3)1

 $AC^2 = BC^2$ Since (sides of equilateral triangle)  $(x-2)^2 + (y-0)^2 = (x-5)^2 + (y-0)^2$ or,  $x^2 + 4 - 4x + y^2 = x^2 + 25 - 10x + y^2$ 6x = 21or,  $x = \frac{7}{2}$ 1 And  $(x-2)^2 + (y-0)^2 = 9$  $\left(\frac{7}{2}-2\right)^2 + y^2 = 9$ or,  $\frac{9}{4} + y^2 = 9$  or,  $y^2 = 9 - \frac{9}{4}$ or,  $y^2 = \frac{27}{4}$  or  $y = \pm \frac{3\sqrt{3}}{2}$ or, (+ve sign to be taken)

Hence, 
$$C = \left(\frac{7}{2}, \frac{3\sqrt{3}}{2}\right)$$
.

Q. 12. If the point C(-1, 2) divides internally the line segment joining the points A(2, 5) and B(x, y) in the ratio 3 : 4, find the value of  $x^2 + y^2$ .

A [Foreign Set I, II, III, 2016]

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Sol.  $\frac{AC}{CB} = \frac{3}{4}$ Given that, Applying section formula for x co-ordinate,  $-1 = \frac{3x+4(2)}{3+4}$ -7 = 3x + 8or, x = -5or Similarly for y co-ordinate,  $2 = \frac{3y + 4(5)}{3 + 4}$ 14 = 3y + 20y = -2or ∴ (x, y) is (-5, -2)Hence,  $x^2 + y^2 = (-5)^2 + (-2)^2$ = 25 + 4[CBSE Marking Scheme, 2016]

Q. 13. If the co-ordinates of points A and B are (-2, -2)and (2, - 4) respectively, find the co-ordinates of P such that  $AP = \frac{3}{\pi}AB$ , where P lies on the line

segment AB. U [CBSE OD, 2015, Set I, II, III, 2015]

Sol.  

$$AP = \frac{3}{7}AB \text{ or, } AP : PB = 3 : 4 \text{ 1}$$

$$P(x, y)$$

$$A(-2, -2)$$

$$3 : 4$$

$$B(2, -4)$$

$$B(2, -4)$$

$$B(2, -4)$$

$$B(2, -4)$$

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$y = \frac{my_2 + ny_1}{m + n}$$

$$x = \frac{3 \times 2 + 4 \times -2}{3 + 4} = -\frac{2}{7}$$

$$y = \frac{3 \times -4 + 4 \times -2}{3 + 4} = -\frac{20}{7}$$

$$y = \frac{3 \times -4 + 4 \times -2}{3 + 4} = -\frac{20}{7}$$

$$y = \frac{20}{7}$$

$$y = \frac{20}{7}$$

and

and

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Q. 14. The co-ordinates of the vertices of  $\triangle ABC$  are A(7, 2), B(9, 10) and C(1, 4). If E and F are the mid-points of AB and AC respectively, prove that  $EF = \frac{1}{2}BC.$ A [CBSE Board Term-2, 2015]

**Sol.** Let the mid-points of AB and AC be  $E(x_1, y_1)$  and  $F(x_2, y_2)$ 

∴ Co-ordinates of point 
$$E = \left(\frac{9+7}{2}, \frac{10+2}{2}\right)$$
  
 $E(x_1, y_1) = (8, 6)$  ½  
A (7, 2)  
A (7, 2)  
B (9, 10)  
C (1, 4)  
Co-ordinates of point  $F = \left(\frac{7+1}{2}, \frac{2+4}{2}\right)$   
 $(x_2, y_2) = (4, 3)$  ½  
Length of  $EF = \left|\sqrt{(8-4)^2 + (6-3)^2}\right|$   
 $= \left|\sqrt{(4)^2 + (3)^2}\right|$   
 $= 5 \text{ units}$  ...(i) 1  
Length of  $BC = \left|\sqrt{(9-1)^2 + (10-4)^2}\right|$   
 $= 10 \text{ units}$  ...(ii)  
From equations (i) and (ii), we get  
 $EF = \frac{1}{2}BC$ . Hence proved. 1

### [CBSE Marking Scheme, 2015]

Q. 15. Find the ratio in which the line segment joining the points A(3, -3) and B(-2, 7) is divided by X-axis. Also find the co-ordinates of point of division. A [CBSE Delhi Term-2, 2015]

$$\frac{1(3)+k(-2)}{1+k} \qquad \qquad 1$$

$$x = \frac{3 - 2 \times \frac{3}{7}}{1 + \frac{3}{7}} = \frac{21 - 6}{7 + 3} = \frac{3}{2}$$

 $\therefore$  Co-ordinates of point P are  $\left(\frac{3}{2}, 0\right)$ . 1

=

[CBSE Marking Scheme, 2014]

## Long Answer Type Questions

 $0 = \frac{1(-3) + k(7)}{1+k}$  $k = \frac{3}{7}$ 

 $x = \frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}$ 

(x, 0)

Ratio = 3:7

► B (-2, 7)

1

AIQ. 1. Find the ratio in which the Y-axis divides the line segment joining the points (-1, -4) and (5, -6). Also find the coordinates of the point of intersection. A [CBSE OD Set-III, 2019]

Sol. Any point on Y-axis is 
$$P(0,y)$$
  
Let P divides AB in  $k:1$   
A(-1, -4)  $P(0, y)$  B(5, -6)  
 $(0, -5k-1) \rightarrow k-1$  i.e. 1:5

$$\Rightarrow \qquad 0 = \frac{5k-1}{k+1} \Rightarrow k = \frac{1}{5} \text{ i.e., } 1:5$$

$$\Rightarrow \qquad y = \frac{-6k-4}{k+1} = \frac{-\frac{6}{5}-4}{\frac{1}{5}+1} = \frac{-26}{6} = \frac{-13}{3} \qquad 1$$

$$\Rightarrow P \text{ is } (0, \frac{-13}{3}) \quad [CBSE Marking Scheme, 2019] \mathbf{1}$$

AIQ. 2. If P(9a - 2, -b) divides the line segment joining<br/>A(3a + 1, -3) and B(8a, 5) in the ratio 3 : 1. Find the<br/>values of a and b.AACBSE SQP, 2016

Sol. By section formula

$$\partial a - 2 = \frac{3(8a) + 1(3a + 1)}{3 + 1}$$
 ...(i) 1

...(ii) 1

1

and

Sol.

A ◀ (3, −3)

or,

or,

Also,

Let the ratio be k : 1,

Using section formula, we get,

From (ii),

$$-b = \frac{15-3}{4} = 3$$

 $-b = \frac{3(5)+1(-3)}{2+1}$ 

$$b = -3$$
$$9a - 2 = \frac{24a + 3a + 1}{4}$$

$$4 1 1 1 4(9a-2) = 27a + 1 36a - 8 = 27a + 1 9a = 9 a = 1 1 1 [CBSE Marking Scheme, 2016]$$

## 5 marks each

Q. 3. The base BC of an equilateral triangle ABC lies on *y*-axis. The co-ordinates of point C are (0, - 3). The origin is the mid-point of the base. Find the co-ordinates of the point A and B. Also find the co-ordinates of another point D such that BACD is a rhombus. A [Foreign Set I, II, 2015]

**Sol.** Co-ordinates of point B are (0, 3)  $\frac{1}{2}$  $\therefore BC = 6$  unit

Let the co-ordinates of point A be (x, 0).  $\frac{1}{2}$ 

 $AB = \left| \sqrt{x^2 + 9} \right|$ 

or,

·:· .·.

$$AB^2 = BC^2$$
$$r^2 + 9 = 36$$

$$x' \leftarrow D$$
  
 $C = 0$   
 $(0, -3)$ 

or,

Co-ordinates of point  $A = (3\sqrt{3}, 0)$ 

Since ABCD is a rhombus.

or, 
$$AB = AC = CD = DB$$

 $\therefore$  Co-ordinates of point  $D = (-3\sqrt{3}, 0)$ . 1

[CBSE Marking Scheme, 2015]

 $x^2 = 27 \text{ or}, x = \pm 3\sqrt{3}$ 

Q. 4. (1, -1), (0, 4) and (-5, 3) are vertices of a triangle. Check whether it is a scalene triangle, isosceles triangle or an equilateral triangle. Also, find the length of its median joining the vertex (1, -1) the mid-point of the opposite side.

A [CBSE, Term-2, 2015]

1



Let the vertices of  $\triangle ABC$  be A(1, -1), B(0, 4) and C(-5, 3). 1

: Using distance formula,

$$AB = \left| \sqrt{(1-0)^2 + (-1-4)^2} \right| \\ = \left| \sqrt{1+5^2} \right| \\ = \sqrt{26} \text{ unit}$$

$$BC = \left| \sqrt{(-5-0)^2 + (3-4)^2} \right|$$
$$= \left| \sqrt{25+1} \right| = \sqrt{26} \text{ unit} \qquad \frac{1}{2}$$
$$AC = \left| \sqrt{(-5-1)^2 + (3+1)^2} \right|$$

 $= \left| \sqrt{36 + 16} \right| = \sqrt{52}$ 

$$= 2\sqrt{13}$$
 unit  $\frac{1}{2}$ 

or, 
$$AB = BC \neq AC$$

or,  $\triangle$ ABC is isosceles.

Now, using mid-point formula, the co-ordinates of mid-point of BC are

$$x = \frac{-5+0}{2} = -\frac{5}{2}$$

$$y = \frac{3+4}{2} = \frac{7}{2}$$

$$D(x, y) = \left(-\frac{5}{2}, \frac{7}{2}\right)$$
1

: Length of median, AD

or,

.:. L

 $\frac{1}{2}$ 

$$= \sqrt{\left(\frac{-5}{2} - 1\right)^2 + \left(\frac{7}{2} + 1\right)^2}$$
$$= \sqrt{\left(\frac{-7}{2}\right)^2 + \left(\frac{9}{2}\right)^2}$$
$$= \sqrt{\frac{130}{4}} = \frac{\sqrt{130}}{2} \text{ unit}$$
ength of median AD is  $\frac{\sqrt{130}}{2}$  units. 1

[CBSE Marking Scheme, 2015]

## TOPIC - 2 Area of Triangle

**Revision Notes** 

> If A( $x_1$ ,  $y_1$ ), B( $x_2$ ,  $y_2$ ) and C( $x_3$ ,  $y_3$ ) are vertices of a triangle, then the co-ordinates of centroid are

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

▶ If  $A(x_1, y_1)$ ,  $B(x_2, y_2)$  and  $C(x_3, y_3)$  are vertices of a triangle, then

Area of 
$$\triangle ABC = \frac{1}{2} \left[ [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \right]$$

> If the points are collinear, then the area of triangle is zero.

# How is it done on the GREENBOARD?

Q.1. Find k, if the point A(2, 3), B(5, K) and C(7, 9) are collinear. Solution: Step 1: If points A, B, and C are collinear then the area of triangle formed by them is zero. Step 2: Area of triangle formed by the vertices  $(x_1, y_1), (x_2, y_2)$  and  $(x_3, y_3)$  is given by  $A = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1)]$ 

 $+ x_3(y_1 - y_2)]$ 

Step 3: According to question,

$$0 = \frac{1}{2} [2(k-9) + 5(9-3)]$$

+ 7(3 - k)or, 2k - 18 + 45 - 15 + 21 - 7k = 0or, -5k + 33 = 0or, 5k = 33or,  $k = \frac{33}{5}$ 

## Very Short Answer Type Questions

Q. 1. If the points A(3, 1), B(5, *p*) and C(7, - 5) are collinear, then find the value of *p*.
 A [CBSE Delhi Set-I, 2020]

**Sol.** Here,  $x_1 = 3$ ,  $x_2 = 5$ ,  $x_3 = 7$  and  $y_1 = 1$ ,  $y_2 = p$ ,  $y_3 = -5$ If points are collinear, then area of triangle = 0

 $\therefore \quad \frac{1}{2} | [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] | = 0 \frac{1}{2}$ 

$$\Rightarrow \frac{1}{2} |[3(p+5) + 5(-5-1) + 7(1-p)]| = 0$$
  
$$\Rightarrow \frac{1}{2} |[3n + 15 - 30 + 7 - 7n]| = 0$$

$$\Rightarrow \qquad \frac{1}{2} |[5p + 15 - 50 + 7 - 7p]| = 0$$
$$\Rightarrow \qquad -4p - 8 = 0$$
$$-4p = 8$$

$$p = -2.$$

 $\frac{1}{2}$ 

Q. 2. If the points (0, 0), (1, 2) and (x, y) are collinear, then find the relation between x and y. [CBSE Term-2, 2015, 2016]

 $\Rightarrow$ 

**Sol.** The points are collinear, then area of triangle = 0

$$\therefore \quad \frac{1}{2} \left| \left[ x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right] \right| = 0$$

## Short Answer Type Questions-I

Q. 1. Show that the points A(0, 1), B(2, 3) and C(3, 4) are collinear.

**Sol.** Area of the triangle formed by the given points A(0, 1), B(2, 3) and C(3, 4)

or, 
$$\frac{1}{2} |[0(2-y) + 1(y-0) + x(0-2)]| = 0$$
  
or,  $\frac{1}{2} |[y-2x]| = 0$   
or,  $2x-y = 0$   
 $\therefore \qquad x = \frac{y}{2} 1$ 

[CBSE Marking Scheme, 2016]

Q. 3. If the points A(x, 2), B(-3, -4) and C(7, -5) are collinear, then find the value of x.

U [Foreign Set-I, II, III, 2015]

2 marks each

**Sol.** Since, the points are collinear, then Area of triangle = 0

$$\frac{1}{2} |[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]| = 0 \qquad \frac{1}{2}$$
$$\frac{1}{2} |[x(-4+5) + (-3)(-5-2) + 7(2+4)]| = 0$$
$$x + 21 + 42 = 0$$

 $x = -63 \frac{1}{2}$ 

## 1 mark each

$$= \frac{1}{2} |0(3-4) + 2(4-1) + 3(1-3)| \quad \mathbf{1}$$
$$= \frac{1}{2} |0 + (2)(3) + (3)(-2)|$$
$$= \frac{1}{2} |6-6|$$
$$= \frac{1}{2} (0)$$
$$= 0$$

... The given points are collinear. 1 [CBSE Marking Scheme, 2016]

Q. 2. Prove that the points (2, -2), (-2, 1) and (5, 2) are the vertices of a right angled triangle. Also find the area of this triangle. A [Foreign Set I, II, III, 2016]



### = 9 + 16= 25 $AC^2 = 25 \text{ or } AC = 5$ $AB^2 + AC^2 = BC^2$ or, Clearly 1 25 + 25 = 50Hence, the triangle is right angled, Area of $\triangle ABC = \frac{1}{2} \times Base \times Height$

$$= \frac{1}{2} \times 5 \times 5 = \frac{25}{2}$$
 sq units. 1

[CBSE Marking Scheme, 2016]

Q. 3. Find the relation between x and y, if the points *A*(*x*, *y*), *B*(– 5, 7) and *C*(– 4, 5) are collinear.

A [CBSE Term,-2, 2015]

Sol. If area covered by the given points is O, the points are collinear. ( A ADC

Area of 
$$\Delta ABC = 0$$
  

$$\frac{1}{2} |[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]| = 0 \qquad \frac{1}{2}$$

$$= \frac{1}{2} |[x(7 - 5) - 5(5 - y) - 4(y - 7)]| = 0 \qquad 1$$
or,
$$2x - 25 + 5y - 4y + 28 = 0 \qquad \frac{1}{2}$$
or,
$$2x + y + 3 = 0$$

[CBSE Marking Scheme, 2015]

3 marks each

## **Short Answer Type Questions-II**

**AI** Q. 1. Find the area of triangle PQR formed by the points P(-5, 7), Q(-4, -5) and R(4, 5). **A** [CBSE Delhi Set-I, 2020] Sol. Here  $x_1 = -5$ ,  $x_2 = -4$ ,  $x_3 = 4$  and  $y_1 = 7$ ,  $y_2 = -5$ ,  $y_3 = 5$ :. Area of  $\triangle PQR = \frac{1}{2} | [x_1(y_2 - y_3) + x_2(y_3 - y_1)] |$  $+ x_3(y_1 - y_2) ] | \frac{1}{2}$  $= \frac{1}{2} \mid [-5 \ (-5 \ -5) \ -4 \ (5 \ -7)$ +4 (7 +5)]| ½  $=\frac{1}{2}\left|\left[-5(-10)-4(-2)+4(12)\right]\right|$  $=\frac{1}{2}|[50+8+48]|$  $\frac{1}{2}$  $=\frac{1}{2} \times 106 = 53$  sq units.  $\frac{1}{2}$ 

 $\mathbf{AI}$  Q. 2. Find the area of triangle ABC with A(1, -4) and the mid-points of sides through A being (2, -1) and C + A [CBSE OD Set-I, 2020] (0, -1).**Sol.** Let the coordinates of the points B and C be (x, y)and (*a*, *b*), then  $\frac{x+1}{2} = 2$ 

$$\Rightarrow x = 4 - 1 = 3 \text{ and } \frac{y - 4}{2} = -1$$
$$\Rightarrow y = -2 + 4 = 2$$

1

or

### **Detailed Solution:**



Q. 3. Two friends Seema and Aditya work in the same office at Delhi. In the Christmas vacations, both decided to go to their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi. Based on the given situation, answer the following questions :

(i) Who will travel more distance, Seema or Aditya, to reach to their hometown ?

- (ii) Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the points represented by Town A and Town B. Find the coordinates of the point represented by the point D.
- (iii) Find the area of the triangle formed by joining the points represented by A, B and C.



[CBSE SQP, 2020]

Sol. (i) A(1, 7), B(4, 2) C(-4, 4) Distance travelled by Seema =  $\sqrt{34}$  units

Distance travelled by Aditya =  $\sqrt{68}$  units 1

(ii) Coordinates of D are  $\left(\frac{1+4}{2}, \frac{7+2}{2}\right) = \left(\frac{5}{2}, \frac{9}{2}\right)$  1

(iii) ar 
$$(\Delta ABC) = \frac{1}{2} \left[ \left[ 1 \left( 2 - 4 \right) + 4 \left( 4 - 7 \right) - 4 \left( 7 - 2 \right) \right] \right]$$

= 17 sq. units 1 [CBSE SQP Marking Scheme, 2020]

### **Detailed Solution:**

According to given graph,

The coordinates, where the station C is situated = (-4, 4)

The coordinates of Town A = (1, 7)

and the coordinates of Town B = (4, 2)

(i) Distance travelled by Seema from station C to their

home town

$$A = \left| \sqrt{(1+4)^2 + (7-4)^2} \right|$$
$$= \left| \sqrt{(5)^2 + (3)^2} \right|$$
$$= \sqrt{34} \text{ unit}$$

Distance travelled by Aditya from C to B

$$= \sqrt{(4+4)^2 + (2-4)^2}$$
$$= \sqrt{(8)^2 + (-2)^2}$$
$$= \sqrt{68} \text{ unit}$$

Hence, Aditya travelled more distance.

(ii) We have, D is the mid-point AB. A(1, 7) C(-4, 4) The coordinates of  $D = \left(\frac{1+4}{2}, \frac{7+2}{2}\right)$  $= \left(\frac{5}{2}, \frac{9}{2}\right).$  1

(iii) Here,  $x_1 = 1$ ,  $y_1 = 7$ ,  $x_2 = 4$ ,  $y_2 = 2$ ,  $x_3 = -4$  and  $y_3 = 4$ .

Area of 
$$\Delta ABC$$
  

$$= \frac{1}{2} |[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]|$$

$$= \frac{1}{2} |[1(2 - 4) + 4(4 - 7) + (-4)(7 - 2)]|$$

$$= \frac{1}{2} |[-2 - 12 - 20]|$$

$$= \frac{1}{2} \times |-34| = |-17|$$

ar  $\Delta ABC = 17$  sq. units

Q. 4. If A(- 5, 7), B(- 4, - 5), C(- 1, - 6) and D(4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD. A [CBSE Delhi/OD, 2018]

[CBSE Term-2, 2015]



= 72 sq. units 1 [CBSE Marking Scheme, 2018]

### **COMMONLY MADE ERROR**

 Some candidates use correct formula for finding area of triangle but simplifying in error.

### **Detailed Solution:**

### **Topper Answer, 2018** (rot to scale) A (-5.7) 15) Vertices of guadrilateral ABCD: B (-4,-5) (choice 2) A (5.7) , B(-4,5), c (-1,-6), P(4,5) Anes of guad ABCD. = Gues (ABD + Gaes (ABCP. r (-1.-6) D area & ABD-> (41,5) = $\frac{1}{2} \left[ x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) \right]$ sq. units. = 1 [-5(-5-5) + (-4) (5-7) + 4(7+5)] = 1 [# so + 8+ 48] - 1 [58+48]. $=\frac{1}{2} \times 106 = 53$ units<sup>2</sup>. anea A BCD = 1 [x1(42-43)+x2(43-41)+x3(41-42)] $=\frac{1}{2}\left[-4\left(-(-5)+(-1)\right)(5+5)+(4)(-4+1)\right]$ = 12 [44-10+4] \$4. Anen of quadhilateral = Anen of two triangles = 57+19=72 units. Area of quadrilateral ABCD is 72 squarts 3

Sol.

**AI** Q. 5. Find the value of k for which the points (3k - 1, k - 2) (k, k - 7) and (k - 1, -k - 2) are collinear.

A [CBSE SQP, 2018]

**Sol.** For collinearity of the points, area of the triangle formed by given points is zero.

$$\frac{1}{2} \{(3k-1) (k-7+k+2) + k(-k-2-k+2) + (k-1)(k-2-k+7)\} = 0$$
$$\frac{1}{2} \{(3k-1) (2k-5) - 2k^2 + 5k - 5\} = 0$$
$$4k^2 - 12k = 0$$

k = 0, 3 1

1

1

If k = 0 then two points are coincide

*.*..

k = 3.

### [CBSE Marking Scheme, 2018]

Q. 6. If the area of triangle with vertices (*x*, 3), (4, 4) and (3, 5) is 4 square units, find *x*. U [CBSE SQP, 2018]

Given,  

$$Ar (ABC) = 4$$

$$\frac{1}{2} |[x(4-5) + 4(5-3) + 3(3-4)]| = 4$$

$$(-x+5) = 8$$

$$-x+5 = 8$$

$$x = -3$$
If
$$-(-x+5) = 8$$

$$x = 13$$

$$\frac{1}{2}$$

Q. 7. If the points A(0, 1), B(6, 3) and C(x, 5) are the vertices of a triangle, find the value of x such that area of  $\triangle ABC = 10$ . [CBSE S.A.II 2016] [CBSE Compt. Set-I, II, III-2018]

**Sol.** Given, area of  $\triangle ABC = 10$ 

ANSWERING TIP

For simplifying, be careful.

$$\therefore \frac{1}{2} |[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]| = 10^{-1/2}$$

Here, 
$$x_1 = 0$$
,  $y_1 = 1$ ,  $x_2 = 6$ ,  $y_2 = 3$ ,  $x_3 = x$  and  $y_3 = 5$   
 $\frac{1}{2}$ 

Then, 
$$\frac{1}{2} |[0(3-5) + 6(5-1) + x(1-3)]| = 10 \frac{1}{2}$$
  
 $\Rightarrow \text{ If } \frac{1}{2} |[0+24+(-2)x]| = 10 \frac{1}{2}$   
 $\Rightarrow -(-2x+24) = 20 | -2x+24 = 20$   
 $\Rightarrow 2x = 20+24 | -2x = -4$   
 $\Rightarrow x = 22 | x = +2. 1$ 

**Sol.** Let the vertices of given triangle be A(0, -1), B(2, 1)and C(0, 3).

Then, the coordinates of mid-points are P(1, 0), Q(1, 2) and R(0, 1). 1 Area of  $\Delta PQR$ ,

$$= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$
  
=  $\frac{1}{2} |(2 - 1) + 1(1 - 0) + 0(0 - 2)|$   
=  $\frac{1}{2} |1 + 1 + 0|$   
= 1 so unit

= 1 sq. unit

Q.9. The area of a triangle is 5 sq. units. Two of its vertices are (2, 1) and (3, -2). If the third vertex is  $\left(\frac{7}{2}, y\right)$ , find the value of y.

### A [CBSE, Delhi Set II, 2017]

1

*:*..

**Sol.** Given, ar  $\triangle ABC = 5$  sq. untis

or, 
$$\frac{1}{2} \left| 2(-2-y) + 3(y-1) + \frac{7}{2}(1+2) \right| = 5$$
 1<sup>1</sup>/<sub>2</sub>

or, 
$$\frac{1}{2} \left| -4 - 2y + 3y - 3 + \frac{21}{2} \right| = 5$$

or, 
$$\left| y + \frac{7}{2} \right| = 10$$
 1

 $\frac{13}{2}$ or, y = 10 -

If 
$$y + \frac{7}{2} = -10$$
 or  $y = -10 - \frac{7}{2} = -\frac{27}{2}$   
Hence, the value of  $y = \frac{13}{2}$  or  $-\frac{27}{2}$  <sup>1/2</sup>  
[CBSE Marking Scheme, 2017]

Q. 10. If  $a \neq b \neq 0$ , prove that the points  $(a, a^2)$ ,  $(b, b^2)$  and (0, 0) will not be collinear.

### U [CBSE, Delhi Set I, II, III, 2017]

Sol. If the area covered by the given points is zero, then the points are collinear.

. Area = 
$$\frac{1}{2} \Big[ a \Big( b^2 - 0 \Big) + b \Big( 0 - a^2 \Big) + 0 \Big( a^2 - b^2 \Big) \Big]$$
  
=  $\frac{1}{2} \Big[ a b^2 - a^2 b + 0 \Big]$  2  
=  $\frac{1}{2} \Big[ a b (b - a) \Big] \neq 0$ 

 $(a \neq b \neq 0)$ 

Hence, the given points are not collinear.

[CBSE Marking Scheme, 2017]

Q. 11. The points A(4, - 2), B(7, 2), C(0, 9) and D(- 3, 5) form a parallelogram. Find the length of altitude of the parallelogram on the base AB.

**U** [CBSE SQP, 2017]

1

1

**Sol.** Let the height of parallelogram taking *AB* as base be *h*.

$$AB = \left| \sqrt{(7-4)^2 + (2+2)^2} \right|$$
  
=  $\left| \sqrt{3^2 + 4^2} \right| = \left| \sqrt{9+16} \right|$   
= 5 units.

Ar  $\triangle ABC$ 

$$= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_2) + x_3(y_1 - y_2)|$$
  

$$= \frac{1}{2} |4(2 - 9) + 7(9 + 2) + 0(-2 - 2)|$$
  

$$= \frac{1}{2} |-28 + 77|$$
  

$$= \frac{1}{2} \times 49 = \frac{49}{2} \text{ sq. units}$$
  
Now,  $\frac{1}{2} \times AB \times h = \frac{49}{2}$   
or,  $5 \times h = 49$ 

or, 
$$h = \frac{49}{5} = 9.8$$
 units. 1

[CBSE Marking Scheme, 2017]

## Long Answer Type Questions

5 marks each

Q. 1. If the points A(k + 1, 2k), B(3k, 2k + 3) and C(5k - 1, 5k) are collinear, then find the value of k.

$$\begin{array}{c} \hline \textbf{CBSE OD, Set-II, 2017} \end{array} \\ \hline \hline \textbf{P} \end{array} \\ \hline \textbf{The points A B and C are collineal} \\ \hline A (ABC) = 0. \\ \hline USing area formula. \\ \hline a_1 = k+1, \quad a_2 = 3k, \quad a_3 = 5k-1 \\ \hline y_1 = 2k, \quad y_2 = 2k+3, \quad y_3 = 5k. \\ \hline Using area formula. \\ \hline a_1(y_2-y_3) + a_2(y_2-y_3) + a_3(y_1-y_2) = 0. \\ \hline (k+1)(2k+3-5k) + 3k(5k-2k) + (5k-1)(2k-2k-3) = 0 \\ \hline (k+1)(2k+3-5k) + 3k(2k) + (5k-1)(-3) = 0. \\ \hline 3(1+k)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline 3(1+k)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline 3(1+k)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(9k) - 3(5k-1) = 0. \\ \hline (k+2)(1-k) + 3(k)(1-k) = 0. \\ \hline (k+2)(1-k)(1-k) = 0. \\$$

Q. 2. If the co-ordinates of two points are A(3, 4), B(5, - 2) and a point P(x, 5) is such that PA = PB, then find the area of  $\triangle PAB$ .

A [CBSE OD Comptt. Set-I, 2017]

**Sol.** Given PA = PB or  $PA^2 = PB^2$ , using distance formula,

$$(x-3)^2 + (5-4)^2 = (x-5)^2 + (5+2)^2$$
 1

On solving, we get, 
$$x = 16$$
 1

$$\therefore$$
 ar  $\Delta PAB$ 

$$= \frac{1}{2} [16(4+2) + 3(-2-5) + 5(5-4)]$$
 1

$$=\frac{1}{2}[96-21+5]=40$$

Hence, area of triangle = 40 sq. units 1 [CBSE Marking Scheme, 2017]

### **Detailed Solution:**

The coordinates of P, A and B are (x, 5), (3, 4) and (5, – 2) respectively, then PA = PB (given)  $\frac{1}{2}$   $\left| \sqrt{(x-3)^2 + (5-4)^2} \right| = \left| \sqrt{(x-5)^2 + (5+2)^2} \right|$  $\Rightarrow \quad \left| \sqrt{(x-3)^2 + 1} \right| = \left| \sqrt{(x-5)^2 + 49} \right|$ 

(By using distance formula) ½ Squaring on both sides, we get

$$(x-3)^{2} + 1 = (x-5)^{2} + 49$$

$$\Rightarrow (x-3)^{2} - (x-5)^{2} = 48$$

$$\Rightarrow [(x-3) + (x-5)][(x-3) - (x-5)]$$

$$= 48 [\because a^{2}-b^{2} = (a+b)(a-b)]$$

$$\Rightarrow (2x-8)(2) = 48$$

$$\Rightarrow 2x - 8 = 24$$

$$\Rightarrow 2x = 32$$

$$\Rightarrow x = 16$$
Now, the point P(x, 5) is P(16, 5)   

$$\therefore \text{ Area of } \Delta PAB = \frac{1}{2} |16(4+2) + 3(-2-5) + 5(5-4)|$$

$$= \frac{1}{2} |96 - 21 + 5| = 40$$

$$\frac{1}{2} |26 - 21 + 5| = 40$$

 $= \frac{1}{2} |96 - 21 + 5| = 40$  <sup>1</sup>/<sub>2</sub>  $\Delta PAB = 40$  sq. units. 1

Hence, area of  $\triangle PAB = 40$  sq. units.

 $\mathbf{AI}$  Q. 3. The co-ordinates of the points A, B and C are (6, 3), (-3, 5) and (4, -2) respectively. P(x, y) is any point in the plane. Show that  $\frac{\operatorname{ar}(\Delta PBC)}{\operatorname{ar}(\Delta ABC)} = \left|\frac{x+y-2}{7}\right|$ 

5

A [Foreign Set I, 2016]

Sol. P(x, y), B(-3, 5), C(4, -2)  
∴ 
$$ar (\Delta PBC) = \frac{1}{2} |x(7) + 3(2 + y) + 4(y - 5)|$$
  
 $= \frac{1}{2} |7x + 7y - 14|$  1½  
 $ar (\Delta ABC) = \frac{1}{2} |6 \times 7 - 3(-5) + 4(3 - 5)|$   
 $= \left|\frac{49}{2}\right|$  1½

$$\therefore \qquad \left| \frac{\operatorname{ar}(\Delta PBC)}{\operatorname{ar}(\Delta ABC)} \right| = \left| \frac{\frac{1}{2}(7x + 7y - 14)}{\frac{49}{2}} \right| \qquad 1$$
$$= \left| \frac{7(x + y - 2)}{49} \right|$$
$$= \left| \frac{x + y - 2}{7} \right| \qquad 1$$

[CBSE Marking Scheme, 2016]

**A** Q. 4. Prove that the area of a triangle with vertices (t, t-2), (t+2, t+2) and (t+3, t) is independent U [CBSE,Delhi Set I, II, III, 2016] of t.

Sol. Area of the triangle 
$$= \frac{1}{2} |t(t+2-t) + (t+2)|$$
  
 $(t-t+2) + (t+3)(t-2-t-2)| 2$   
 $= \frac{1}{2} [2t+2t+4-4t-12] 1$   
 $= 4$  sq. units. 1

1 which is independent of t. 1 [CBSE Marking Scheme, 2016]

Q. 5. In the given figure, the vertices of  $\triangle$ ABC are A(4, 6), B(1, 5) and C(7, 2). A line-segment DE is drawn to intersect sides AB and AC at D and E respectively such that  $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$ . Calculate the area of

 $\triangle$ **ADE** and compare it with area of  $\triangle$ **ABC**.



[CBSE Delhi Board, 2015]

**Sol.** Area of a triangle having vertices  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  is given by,

$$\Delta = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right| \qquad \frac{1}{2}$$

ar 
$$\triangle ABC = \frac{1}{2} |4(5-2) + 1(2-6) + 7(6-5)| \frac{1}{2}$$

ar 
$$\triangle ABC = \frac{1}{2} |12 + (-4) + 7|$$
  
ar  $\triangle ABC = \frac{15}{2}$  sq. units. 1

In  $\triangle$ ADE and  $\triangle$ ABC,

$$\frac{AD}{AB} = \frac{AE}{EC} = \frac{1}{3}$$
<sup>1/2</sup>

 $\angle DAE = \angle BAC$ and (Common)

Hence 
$$\Delta ADE \sim \Delta ABC$$
 (By AAA)

or, 
$$\frac{\text{Area }\Delta ADE}{\text{Area }\Delta ABC} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$
 <sup>1</sup>/<sub>2</sub>

or, 
$$\frac{\operatorname{Ar} \Delta ADE}{\left(\frac{15}{2}\right)} = \frac{1}{9}$$

or, Area 
$$\triangle ADE = \frac{15}{2 \times 9} = \frac{5}{6}$$
 sq. units. 1

Area *ADE* : Area *ABC* = 
$$\frac{5}{6} : \frac{15}{2} = 1 : 9$$
 1

Q. 6. Find the values of k so that the area of the triangle with vertices (1, -1), (-4, 2k) and (-k, -5) is 24 sq. units. A [CBSE Board, 2015]

**Sol.** Area of triangle = 
$$\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_3 - y_1)| + \frac{1}{2} |x_1(y_2 - y_3)| + \frac{1}{2} |x_1($$

$$x_3(y_1 - y_2)$$

or, 
$$24 = \frac{1}{2} |1(2k+5) - 4(-5+1) - k(-1-2k)|$$
 2

$$48 = 2k + 5 + 16 + k + 2k^2 \quad \mathbf{1}$$

or, 
$$2k^2 + 3k - 27 = 0$$
 1

OF

or, 
$$(k-3)(2k+9) = 0$$
  
or,  $k = 3, \ k = \frac{-9}{2}$  1

[CBSE Marking Scheme, 2015]

## Visual Case Based Questions

### 4 marks each

### Note: Attempt any four sub parts from each question. Each sub part carries 1 mark each

**Al** Q. 1. The diagram show the plans for a sun room. It will be built onto the wall of a house. The four walls of the sun room are square clear glass panels. The roof is made using,

- Four clear glass panels, trapezium in shape, all of the same size
- One tinted glass panel, half a regular octagon in shape



(i) Refer to Top View, find the mid-point of the segment joining the points J(6, 17) and I(9, 16).

(a)	$\frac{33}{2}, \frac{15}{2}$	(b)	$\frac{3}{2}, \frac{1}{2}$
(c)	$\frac{15}{2}, \frac{33}{2}$	(d)	$\frac{1}{2}, \frac{3}{2}$

Sol. Correct option: (c).

Explanation: Mid-point of J(6, 17) and I(9, 16) is

$$x = \frac{6+9}{2}$$
 and  $y = \frac{17+16}{2}$   
 $x = \frac{15}{2}$  and  $y = \frac{33}{2}$ 

(ii) Refer to front View, the distance of the point P from the *y*-axis is:
(a) 4 (b) 15

a)	4	(0)	15
c)	19	(d)	25

- Sol. Correct option: (a). Explanation: The distance of the point P from the Y-axis = 4. [CBSE Marking Scheme, 2020] 1
- (iii) Refer to front view, the distance between the points A and S is
  (a) 4
  (b) 8
  (c) 14
  (d) 20

**Sol.** Correct option: (c).

**Explanation:** A's coordinates = (1, 8)

S's coordinates = (15, 8)  
Then, 
$$AS = \left| \sqrt{(15-1)^2 + (8-8)^2} \right|$$

A [CBSE SQP, 2020-21]

$$= \sqrt{(14)^2}$$
  
= 14. 1  
[CBSE Marking Scheme, 2020]

(iv) Refer to front view, find the co-ordinates of the point which divides the line segment joining the points A and B in the ratio 1 : 3 internally.

(a)	(8.5, 2.0)	(b)	(2.0, 9.5)
(c)	(3.0, 7.5)	(d)	(2.0, 8.5)

Sol. Correct option: (d).

**Detailed Solution:** 

1

The coordinates of 
$$A = (1, 8)$$
  
The coordinates of  $B = (4, 10)$   
Also,  $m = 1$  and  $n = 3$   
Then,  $(x, y) = \left(\frac{1 \times 4 + 3 \times 1}{1 + 3}, \frac{1 \times 10 + 3 \times 8}{1 + 3}\right)$   
 $= \left(\frac{7}{4}, \frac{34}{4}\right)$   
 $= (1.75, 8.5)$  1  
(v) Refer to front view, if a point  $(x, y)$  is equidistant  
from the Q(9, 8) and S(17, 8), then  
(a)  $x + y = 13$  (b)  $x - 13 = 0$   
(c)  $y - 13 = 0$  (d)  $x - y = 13$   
Sol. Correct option: (b).  
Explanation:  $x - 13 = 0$ . 1

[CBSE Marking Scheme, 2020]

### **Detailed Solution:**

Let point be P(x, y) $PQ^2 = PS^2$ 

or, 
$$(x-9)^2 + (y-8)^2 = (x-17)^2 + (y-8)^2$$
  
or,  $x-13 = 0$  1

Q. 2. Ayush Starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office.

(Assume that all distances covered are in straight lines). If the house is situated at (2, 4), bank at (5, 8), school at (13, 14) and office at (13, 26) and coordinates are in km.



1

 $= 10 \, \text{km}$ 

(b) 26.4 km (d) 26 km

 $= \sqrt{(13-2)^2 + (26-4)^2}$ 

(iii) What is the distance between house and office?

Explanation: Distance between house to office,

(a) 24.6 km

Sol. Correct option: (b).

(c) 24 km

Q. 3. In order to conduct Sports Day activities in your School, lines have been drawn with chalk powder at a distance of 1 m each, in a rectangular shaped ground ABCD, 100 flowerpots have been placed at a distance of 1 m from each other along AD, as shown in given figure below. Niharika runs 1/4<sup>th</sup> the distance AD on the 2<sup>nd</sup> line and posts a green flag. Preet runs 1/5<sup>th</sup> distance AD on the eighth line and posts a red flag.



Mid-point of line segment joining the green and red flags

$$= \left(\frac{2+8}{2}, \frac{25+20}{2}\right)$$
$$= (5, 22.5)$$

(v) If Joy has to post a flag at one-fourth distance from green flag, in the line segment joining the green and red flags, then where should he post his flag ?
(a) (3.5,24)
(b) (0.5,12.5)

(a)	(3.5,24)	(0)
(c)	(2.25.8.5)	(d)

**Sol.** Correct option: (a).

Explanation: Position of Joy's flag

 Mid-point of line segment joining green and blue flags

(25, 20)

$$= \left[\frac{2+5}{2}, \frac{25+22.5}{2}\right]$$

 $= [3.5, 23.75] \sim [3.5, 24]$ 

Q. 4. The class X students school in krishnagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is triangular grassy lawn in the plot as shown in the figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



$$= \frac{1}{2} [-12 + 21]$$
  
=  $\frac{1}{2} [9]$   
= 4.5 sq. units.

(v) Calculate the area of the triangles if C is the origin.

(a)	8		(b)	5
(c)	6.25		(d)	4.5

(c) 6.25 Sol. Correct option: (d).

## **SELF ASSESSMENT TEST - 3**

### Maximum Time: 1 hour

Q. 1. To locate a point Q on line segment AB such that  $BQ = \frac{5}{7} \times AB$ . What is the ratio of line segment in

which AB is divided ?

- A [CBSE, Board Term-2, 2013] 1
- Q. 2. Find the mid-point of P(- 5, 0) and Q (5, 0).
- Q. 3. If the distance between the points (4, p) and (1, 0) is 5, then the value of *p*. **R**1
- Q.4. AOBC is a rectangle whose three vertices are A(0, 3), O(0, 0) and B(5, 0). Find the length of its diagonal. U 1
- O. 5. Find the ratio in which the line segment joining the points (6, 4) and (1, -7) is divided by the *x*-axis. 1

### Q. 6. VISUAL CASE STUDY BASED QUESTIONS:

In a room, 4 friends are seated at the points A, B, C and D as shown in figure. Reeta and Meeta walk into the room and after observing for a few minutes Reeta asks Meeta.



Q. 1. What is the position of A? () (4 0) (1)

Q. 2. What is the middle position of B and C?

(a) 
$$\left(\frac{15}{2}, \frac{11}{2}\right)$$
 (b)  $\left(\frac{2}{15}, \frac{11}{2}\right)$   
(c)  $\left(\frac{1}{2}, \frac{1}{2}\right)$  (d) None of t

(d)

(a) (6, 0) (b)(0, 6)

(c) 
$$(6, 1)$$
 (d)  $(1, 6)$ 

- Q. 4. What is the distance between A and B?
  - (a)  $3\sqrt{2}$  unit (b)  $2\sqrt{3}$  unit
  - (c)  $2\sqrt{2}$  unit (c)  $3\sqrt{3}$  unit
- Q. 5. What is the distance between C and D
  - (a)  $\sqrt{2}$  unit  $2\sqrt{2}$  unit (b)
  - (c)  $3\sqrt{2}$  unit  $4\sqrt{2}$  unit (c)
- Q. 7. If P(2, -1), Q(3, 4), R(-2, 3) and S(-3, -2) be four points in a plane, show that PQRS is a rhombus but not a square. A [CBSE Term-2, 2012] 2
- Q. 8. If (*a*, *b*) is the mid-point of the segment joining the points A(10, - 6) and B(k, 4) and a - 2b = 18, find the value of *k* and the distance AB.

C + A [CBSE Term-2, 2012] 2

None of these

- Q. 9. The co-ordinates of vertices of  $\triangle$ ABC are A(1, -1), B(-4, 6) and C(-3, -5). Draw the figure and prove that  $\triangle ABC$  is a scalene triangle. Find its area also. A [CBSE Term-2, 2014] 3
- Q. 10. If (3, 2) and (-3, 2) are two vertices of an equilateral triangle which contains the origin, find the third vertex. A [CBSE, Term-2, 2012] 3
- Q. 11. A(4, -6), B(3, -2) and C(5, 2) are the vertices of a **ABC** and AD is its median. Prove that the median AD divides  $\triangle ABC$  into two triangles of equal A [CBSE O.D. 2014] 5 areas.